Solving the Restricted Assignment Problem to Schedule Multi-Get Requests in Key-Value Stores

L.-C. Canon, A. Dugois, and L. Marchal

Anthony Dugois

FEMTO-ST, University of Franche-Comté

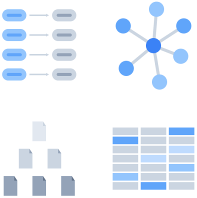
Besançon, France

Euro-Par Conference 2024

Madrid, Spain

August 29, 2024

Applicative Context: Persistent Key-Value Stores



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Wide-Column Databases
In-Memory Key-Value Store
Persistent Key-Value Stores
...

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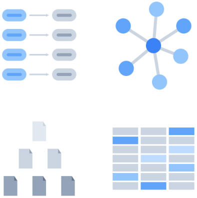

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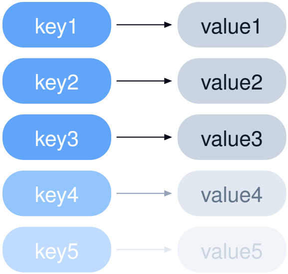

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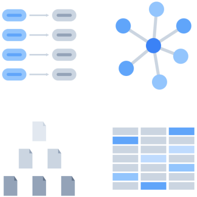

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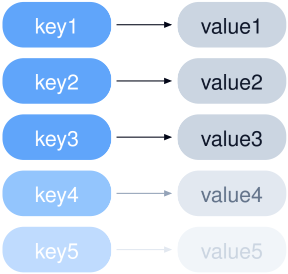
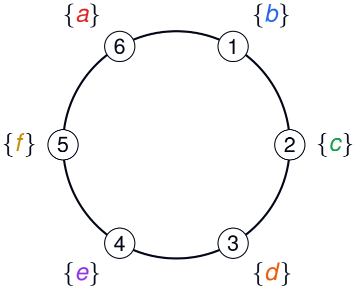


`get(key1)` `put(key2,value2)`

`mget(key1, key2, key4)`

Multi-Get Requests in Distributed Key-Value Stores



- Each server holds a data partition

◯ Servers of the key-value stores

$\{x, y\}$ Data partition of the server

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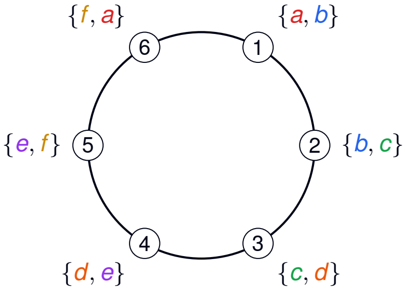

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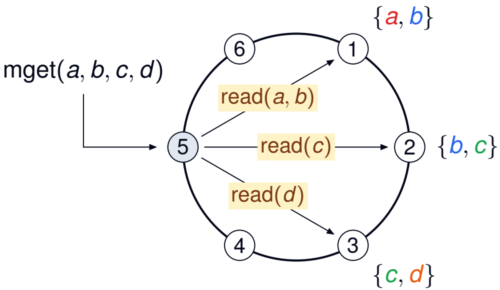

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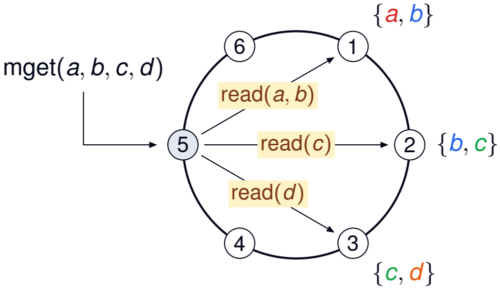

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- Data replicated on k successor servers
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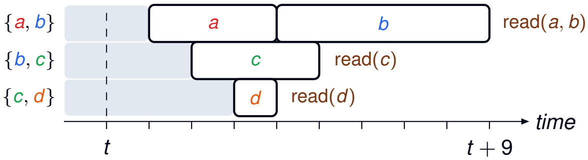
→ Splitting is not trivial: reading different items takes different times, and server loads are different

Multi-Get Request Splitting



$C_{\max} = t + 9$, non-optimal

$C_{\max}$ = maximum completion time of read operations (makespan)

t = arriving time of the multi-get request

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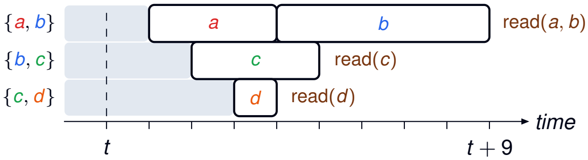

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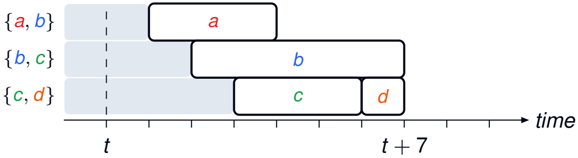

$C_{\max} = t + 7$, optimal

$C_{\max}$ = maximum completion time of read operations (makespan)

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(Almost) Equivalent Scheduling Problem

Restricted Assignment on Intervals (RAI)

- **Input**:
  - n jobs (= read operations)
  - m identical machines (= servers)
  - processing times p_j (= times to read items)
  - intervals of compatible machines $\langle a_j, b_j \rangle = \{a_j, a_j + 1, \cdots, b_j\}$
- **Output**: sets of jobs J_i to put on each machine i
- **Objective**: minimize makespan

Results

- **Existing work**:
  - 2-approximation for $R \,||\, C_{\max}$ [Lenstra et al., 1990]
  - $(2 - 1/2^k)$-approximation for $P \,|\, \mathcal{M}_j \,|\, C_{\max}$ when $p_j \in \{1, 2, \cdots, 2^k\}$ [Biró et al., 2014]
  - No PTAS for RAI unless $P = NP$ [Maack et al., 2020]
  - Optimal algorithm for RAI when $p_j = 1$ and m is fixed [Lin et al., 2004]

$\rightarrow$ Real-time scheduling: cannot afford too complex algorithms

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  $\rightarrow$ Real-time scheduling: cannot afford too complex algorithms

- **In this talk**:

  $\rightarrow$ Efficient $(2 - 1/m)$-approximation for RAI

  $\rightarrow$ Efficient $(4 - 2/m)$-approximation for a generalized version

  $\rightarrow$ Efficient and qualitative heuristics

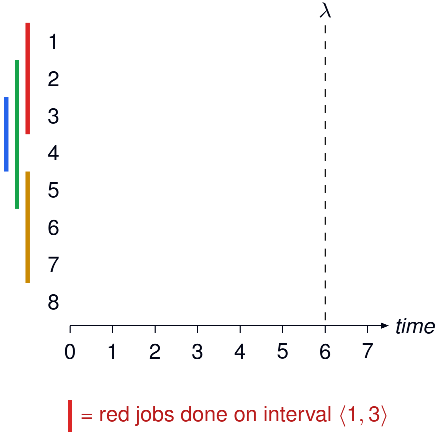
Estimated Least Flexible Job

ELFJ Algorithm

Parameter: estimated makespan λ.

Starting from the first machine, build a schedule that finishes no later than time λ by processing in priority the most constrained job.

Time complexity: $O(n \log n + mn)$ + time to compute λ



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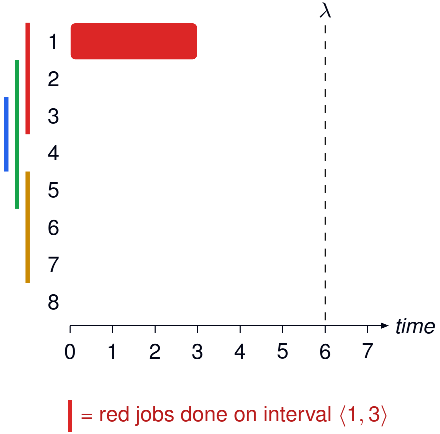

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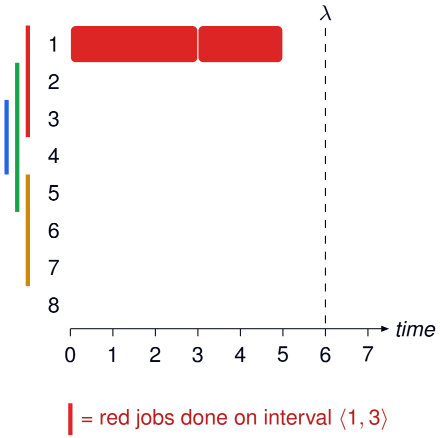

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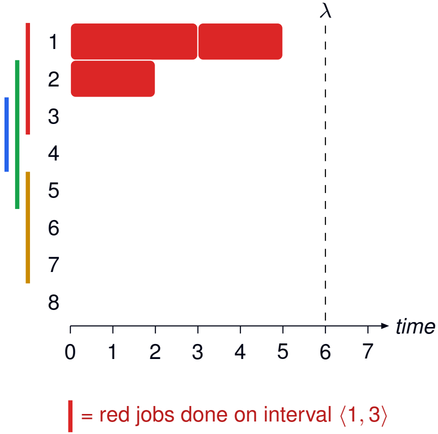

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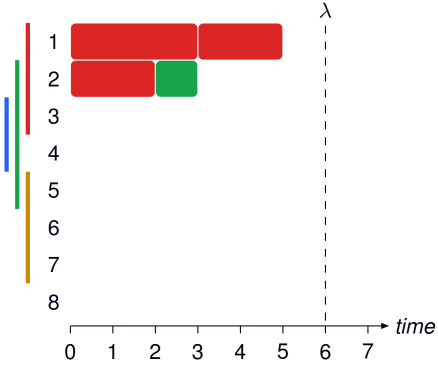

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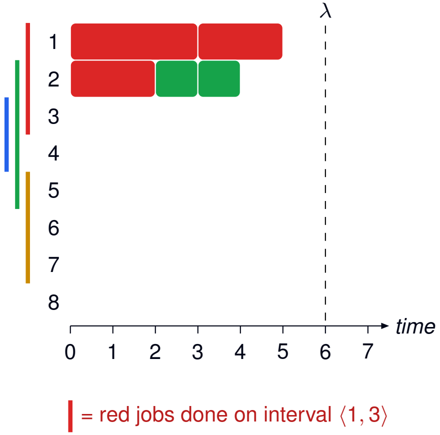

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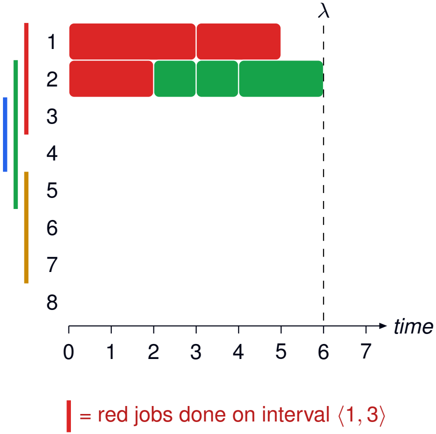

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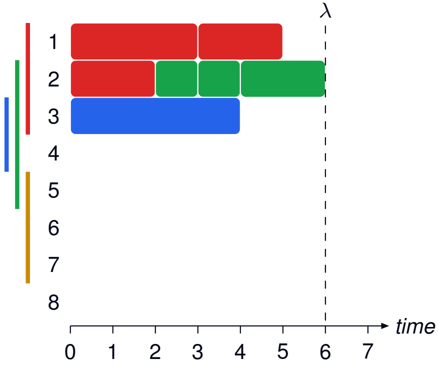

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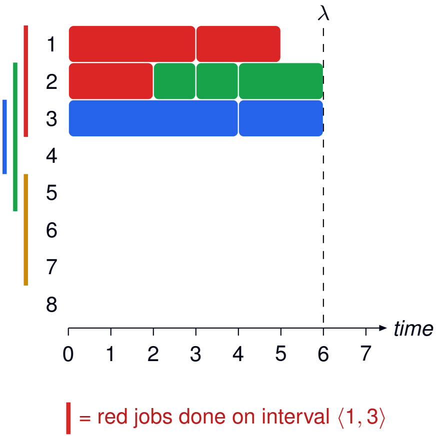

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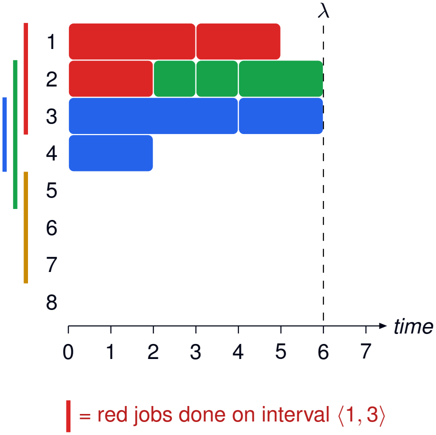

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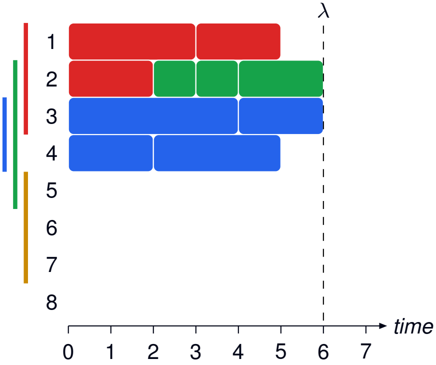

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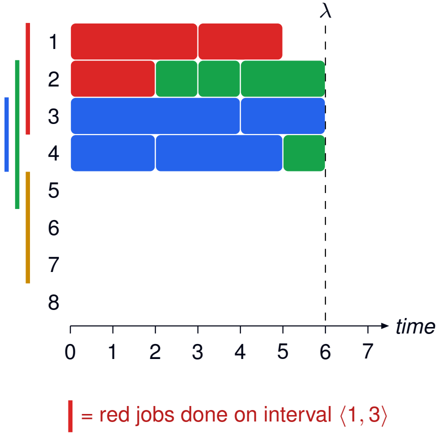

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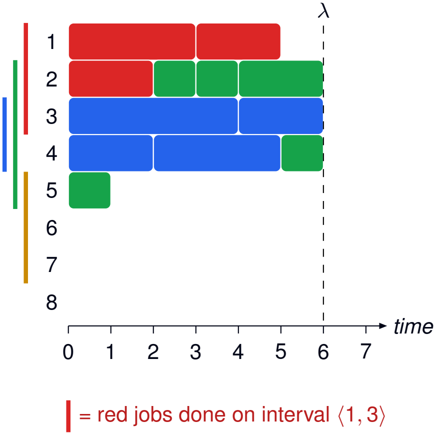

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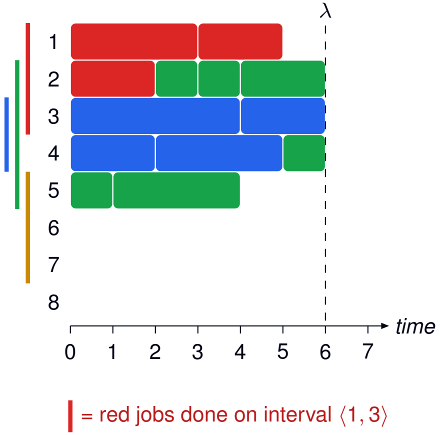

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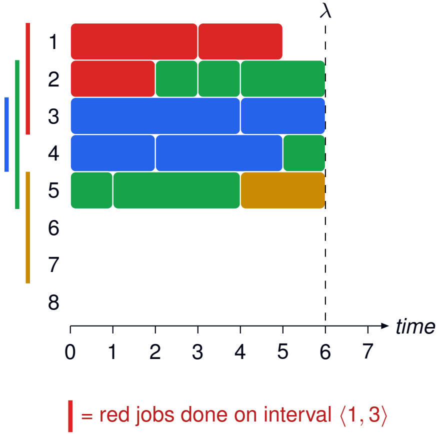

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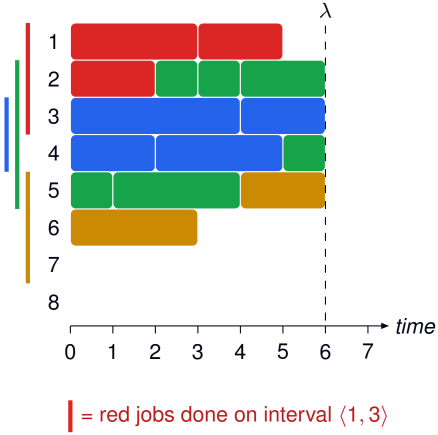

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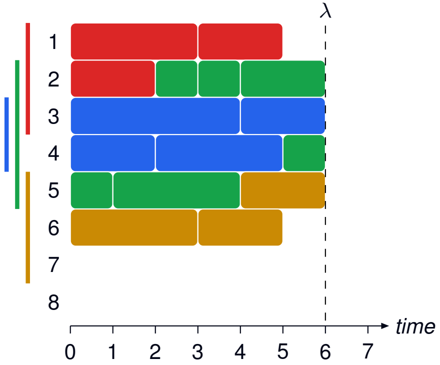

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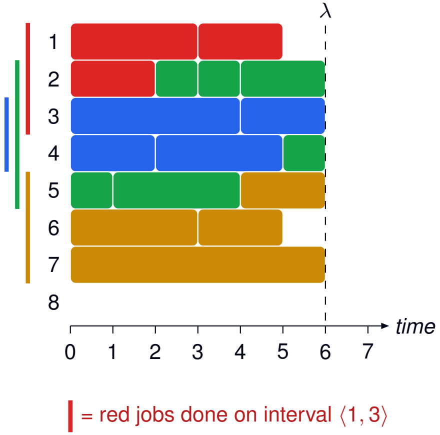

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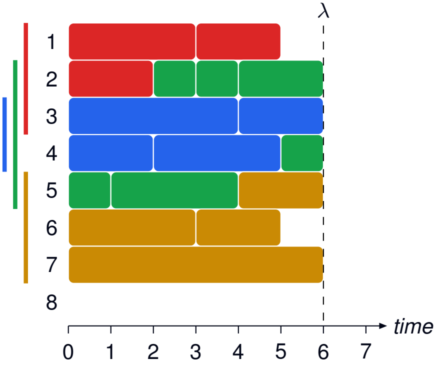

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Result

Computing Makespan λ

- $\lambda = \tilde{w}_{\text{max}} + (1 - 1/m)\, p_{\text{max}}$
- p_{max} = maximum processing time among jobs
- $\tilde{w}_{\text{max}} = \max_{\alpha \leq \beta} \tilde{w}_{\langle \alpha, \beta \rangle}$ where $\tilde{w}_{\langle \alpha, \beta \rangle}$ is the average amount of work that must be done on interval $\langle \alpha, \beta \rangle$

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Theorem

ELFJ is a $(2 - 1/m)$-approximation algorithm for RAI with arbitrary jobs.

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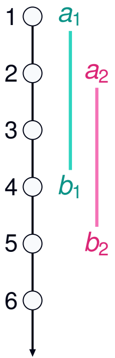
  $\rightarrow$ Can be computed in time $O(m^2 + n)$ with dynamic programming

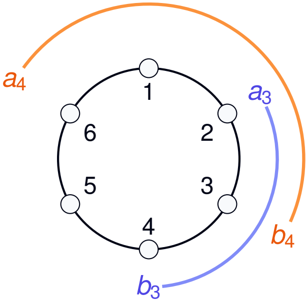
Theorem

ELFJ is a $(2 - 1/m)$-approximation algorithm for RAI with arbitrary jobs.

Runs in time $O(m^2 + n \log n + mn)$.

Generalizing the Problem with Circular Intervals





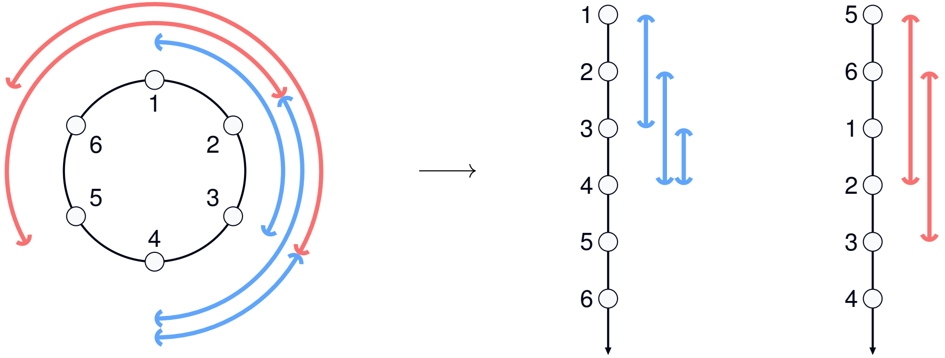
Regular RAI problem

$$\langle \alpha, \beta \rangle = \{\alpha, \alpha + 1, \cdots, \beta\}$$

Circular RAI problem

$$\langle \alpha, \beta \rangle = \begin{cases} \{\alpha, \alpha + 1, \cdots, \beta\} & \text{if } \alpha \leq \beta \\ \{\alpha, \cdots, m\} \cup \{1, \cdots, \beta\} & \text{otherwise.} \end{cases}$$

Splitting the Circular Problem into Regular Sub-Problems



Circular instance $\mathcal{I}$
with all jobs

Regular sub-instance $\mathcal{I}_1$
with *regular* jobs

Regular sub-instance $\mathcal{I}_2$
with *circular* jobs

Result

Double ELFJ Algorithm (DELFJ)

Apply ELFJ for regular jobs, then for the circular ones, and merge schedules.

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Double ELFJ Algorithm (DELFJ)

Apply ELFJ for regular jobs, then for the circular ones, and merge schedules.

Theorem

DELFJ is a $(4 - 2/m)$-approximation algorithm for circular RAI.

Proof sketch:

1. By previous theorem:
   - $\text{ELFJ}(\mathcal{I}_1) \leq (2 - 1/m)\,\text{OPT}(\mathcal{I}_1)$
   - $\text{ELFJ}(\mathcal{I}_2) \leq (2 - 1/m)\,\text{OPT}(\mathcal{I}_2)$
2. Moreover, $\text{OPT}(\mathcal{I}_1) \leq \text{OPT}(\mathcal{I})$ and $\text{OPT}(\mathcal{I}_2) \leq \text{OPT}(\mathcal{I})$
3. Finally, $\text{DELFJ}(\mathcal{I}) \leq \text{ELFJ}(\mathcal{I}_1) + \text{ELFJ}(\mathcal{I}_2) \leq (4 - 2/m)\,\text{OPT}(\mathcal{I})$

Searched Least Flexible Job

SLFJ Algorithm

Progressively searches for a feasible makespan λ by successively applying ELFJ.

```
compute w̃_max as if jobs were unitary
δ ← 0
repeat
    apply ELFJ with λ = ⌈w̃_max⌉ + δ
    δ ← INCREASE(δ)
until all jobs are assigned
```

Note 1: this terminates because it always finds a solution when $\delta \geq p_{\max}$.

Note 2: quality and speed both depend on the INCREASE function.

Heuristics

Two Variants

- Arithmetic SLFJ
  - INCREASE : $\delta \to \delta + 1$
  - Better quality, slower convergence
- Geometric SLFJ
  - INCREASE : $\delta \to \max(1, 2\delta)$
  - OK quality, faster convergence

Simulation Setup

Baseline

- RANDOM: randomly assign each job to a compatible machine
- EFT-MIN: assign each job to the *first* compatible machine that completes it the earliest
- EFT-RAND: same as EFT-MIN, but with a randomized tie-break

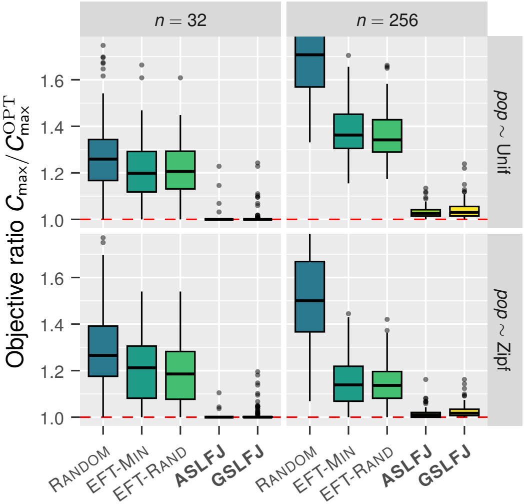
Instance generation

- 48 servers
- Replication factor of 3
- 100 000 keys
- Reading times drawn from exponential distribution
- Two parameters:
  - n = number of read operations in a multi-get request
  - pop = popularity distribution of the keys

Makespan of Each Multi-Get Request

- Ratio between makespan of **each multi-get request** and optimal solution (lower is better)
- Fixed sizes (32 and 256)
- Popularity distributions:
  - Unif: keys have the same probability to be requested
  - Zipf: the probability of a key to be requested is correlated to its rank

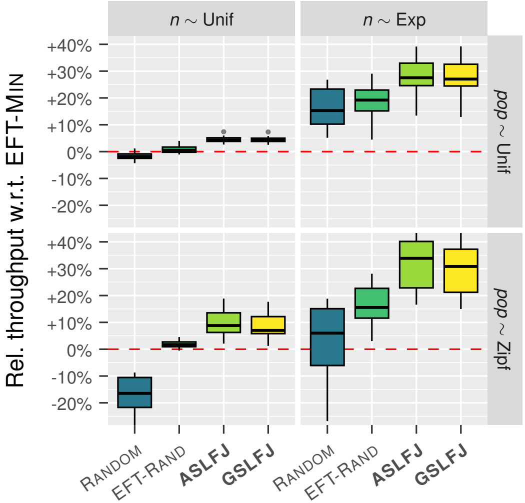
→ GSLFJ converges faster, with a quality almost equal to ASLFJ



Throughput of a Stream of Requests

- Ratio between throughput of **a stream of requests** and solution given by baseline heuristic EFT-MIN (higher is better)
- Size distributions:
  - Unif (between 1 and 256)
  - Exp: “small” requests more probable

→ Optimizing multi-get requests individually leads to global improvements



Conclusion

- $(2 - 1/m)$-approx. for RAI
- Circular (generalized) version of RAI + $(4 - 2/m)$-approx.
- Heuristics give close-to-optimal solutions in practice
- Optimizing individual requests leads to global improvements

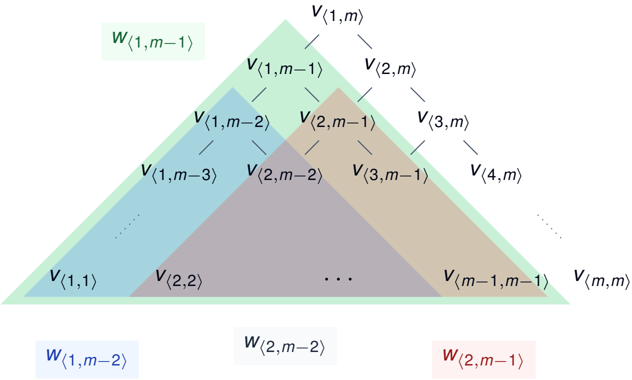
- **Perspectives**:
  - Is there a better approx. for circular RAI?
  - What guarantees can we have if only estimations are available for p_j?
  - Experiment heuristics in actual systems

Thank you!

Read the paper

Reach me at **anthony.dugois@univ-fcomte.fr**

Computing Makespan λ



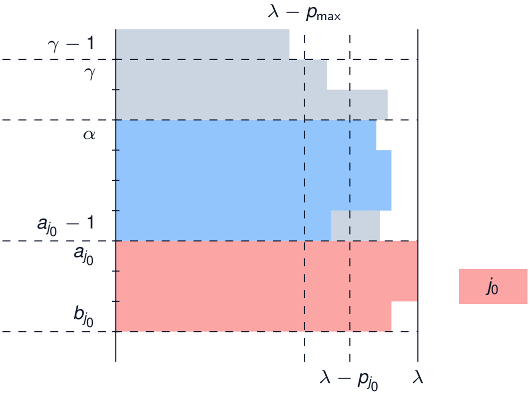
- $v_{\langle \alpha,\beta \rangle}$: total work of jobs j such that $a_j = \alpha$ and $b_j = \beta$.
- $w_{\langle \alpha,\beta \rangle}$: total work of jobs j such that $\alpha \leq a_j \leq b_j \leq \beta$.

Dynamic programming: $w_{\langle \alpha,\beta \rangle} = v_{\langle \alpha,\beta \rangle} + w_{\langle \alpha,\beta-1 \rangle} + w_{\langle \alpha+1,\beta \rangle} - w_{\langle \alpha+1,\beta-1 \rangle}$

Time complexity: $O(m^2 + n)$

Proof Sketch

1. $\lambda = \tilde{w}_{\max} + (1 - 1/m)\, p_{\max}$
2. Suppose by contradiction that a job j_0 cannot be scheduled before λ
3. Progressively search for a machine α such that almost all the work done on $\alpha, \alpha + 1, \cdots, b_{j_0}$ is included in $w_{\langle \alpha, b_{j_0} \rangle}$
   - either all jobs done on $\langle a_{j_0}, b_{j_0} \rangle$ are included in $w_{\langle a_{j_0}, b_{j_0} \rangle}$, or
   - there is a job j_1 done on $\langle a_{j_0}, b_{j_0} \rangle$ such that $a_{j_1} < a_{j_0}$



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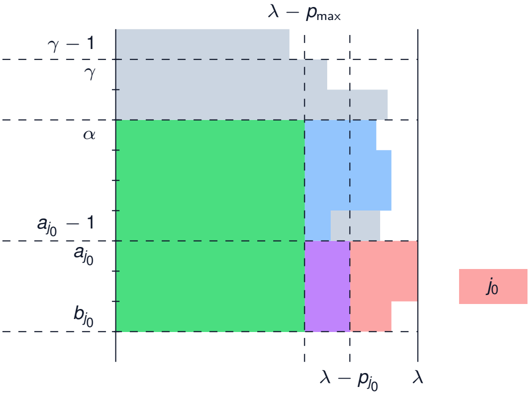
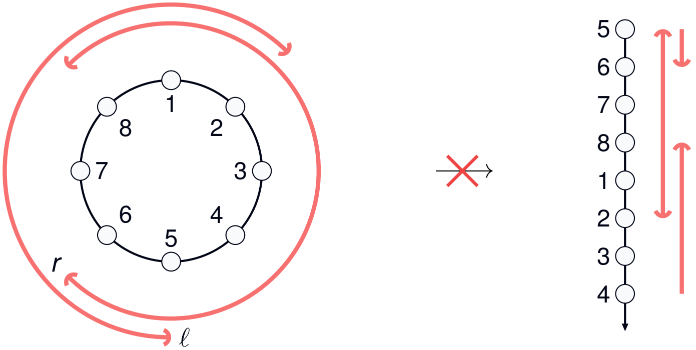
4. $w_{\langle \alpha, b_{j_0} \rangle} > (b_{j_0} - \alpha + 1)(\lambda - p_{\max}) + (b_{j_0} - a_{j_0} + 1)(\lambda - p_{j_0} - (\lambda - p_{\max})) + p_{j_0}$,
5. Leads to $\lambda < \tilde{w}_{\langle \alpha, b_{j_0} \rangle} + (1 - 1/m)\, p_{\max}$
6. $\rightarrow$ Contradicts 1.



Necessary Condition for Splitting



$\mathcal{I}_2$ cannot be regularized if $\ell \leq r$